Nonperturbative Flow Equations at finite Temperature

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QCD incorporates different relevant excitations at different length scales. At short distances the relevant degrees of freedom are quarks and gluons while particles which are observed at large scales are colorless mesons and hadrons. The theoretical challenge is to find a bridge between one set of degrees of freedom (quarks and gluons) to another (mesons and hadrons). As a guiding principle chiral symmetry, which is broken in the vacuum is used. It is believed that the linear σ-model is an effective description for QCD for scales below the mesonic compositeness scale. It also shares an adequate realism with a good practicality in applications.

Recently, renormalization group methods have been applied to calculate the parameters of the σ-model for all resolution scales. The fundamental idea is to follow the dynamics of the system by integrating out quantum fluctuations in infinitesimal intervals from a high momentum scale to the far infrared. In practice these so-called exact renormalization group flow equations have to be truncated in order to obtain a finite dimensional set of differential equations which can be solved numerically. They allow to treat the critical fluctuations of the long range σ-field near the second order chiral phase transition at finite temperature for two massless quark flavors, where mean field methods or sub summations of the effective mass type are insufficient.

In our work [1] we follow the spirit of this renormalization group approach and parametrize the shape of the effective mesonic potential of the linear σ-model, where we only allow quartic and quadratic couplings. This approximation together with a novel heat-kernel infrared cutoff prescription allows us to derive new and very transparent formulae for the evolution equations. Our cutoff function is different from previous work [2] and it allows us to calculate analytically the threshold functions, which arise in the flow equations and describe the smooth decoupling of massive modes from the evolution towards small infrared scales. This work enables us to get analytical insight into the physics inherent in the method of evolution equations.

For high resolution a typical scale associated with perturbative QCD is $\Lambda \geq 1.5$ GeV. RHIC and LHC physics for particles with $p_\perp \geq \Lambda$ will be dominated by such processes. Since the strength of the QCD interactions increases with decreasing momentum transfer, characteristic $\bar{q}q$ bound states will form and influence the dynamics at larger distances. A typical resolution where these processes start to become important is $\Lambda_{\chi SB} \approx 1.0$ GeV. Below this scale chiral symmetry is spontaneously broken and the vaccum acquires a nonvanishing quark and/or meson condensate. We think that our heat-kernel cutoff corresponds to the resolution scale of the photon Q^2 in deep inelastic scattering, where around $Q^2 \approx 1$ GeV^2 the behavior of the structure function F_2 changes qualitatively indicating the phenomenon of chiral symmetry restoration. We use this experimental hint for the transition around 1 GeV as input to our calculation.

In the broken phase the constituent quark mass will increase with decreasing resolution and remain finite down to $\Lambda \approx \Lambda_{QCD}$. In the interval $\Lambda_{QCD} \leq k \leq \Lambda_{\chi SB}$ the dynamics is governed by constituent quarks interacting via pions and σ-mesons. One sees nicely in our approach how the different quantum fluctuations from the σ-mesons and constituent quarks become unimportant when the infrared scale parameter becomes smaller than the respective masses of these states. These modes then decouple from the further evolution, leaving the zero mass pions alone in the evolution.

One also recognizes the different signs of the bosonic and fermionic contributions to the flow equations. The bosons lead to an infrared stable (ultraviolet unstable) coupling, whereas the fermions counteract this tendency. Going from high scales k to low k one sees that the mesonic self-interaction λ_k balances at intermediate values of k, whereas in the far infrared the boson term wins. At $k = k_{\chi SB} \approx 800$ MeV the β-function of λ_k is continuous and the mass parameter m_k of the mesonic potential tends to zero signaling symmetry breaking. The vacuum expectation value ϕ_k stabilizes at small values of k and the evolution ends with $\lim_{k\to 0} \phi_k = f_\pi$.

The $k_{\chi SB}$ scale found in this calculation is somewhat smaller than the resolution Q of the photon in deep inelastic scattering, but this result could be improved by a more sophisticated calculation with a running Yukawa coupling and wave function renormalizations.

After having solved the evolution equations at zero temperature for reasonable starting values of the coupling constants, we pursue the evolution at finite temperature. Here the relevant parameter is the ratio of the temperature over the infrared scale parameter. Decoupling now sets in when the ratio of masses plus the Matsubara frequencies over the infrared scale becomes large. At high temperatures the summation over Matsubara frequencies is dominated by the lowest mode, thereby reducing softly the dynamics to the corresponding three-dimensional field theory, which is the purely bosonic $O(4)$-model. We find in the chiral limit a second order phase transition at a critical temperature $T_c \approx 130$ MeV. Power law behavior is also visible in the order parameter leading to the critical index $\beta = 0.40$ which is in good agreement with results from Monte Carlo calculations.

References

[1] B.-J. Schaefer and H.J. Pirner, *Nonperturbative Flow Equations with Heat-Kernel Methods at finite Temperature* (1997), `hep-ph/9712413`.

[2] J. Berges, D.-U. Jungnickel and C. Wetterich, *Two Flavor Chiral Phase Transition from Nonperturbative Flow Equations* (1997), `hep-ph/9705474`.